UNIVERSITY OF EDUCATION
"UExam" Semester-II, 2019
Msc. Mathematics Session:2018-20

Course Code: MATH3117
Subject: Real Analysis-II

Time Allowed: 100 Minutes.

Max. Marks: 42

Section II (Short Answer)

Q.2- Write short answers of the following. 3x6 = 18

i. Give an example of sequence of functions, which converges point wise but does not converge uniformly.

ii. Let

$$f_n(x)=\begin{cases} n^2x, & \text{for } 0\le x\le \frac{1}{n} \\ -n^2(x-2/n), & \text{for } \frac{1}{n}\le x\le \frac{2}{n} \\ 0, & \text{for } \frac{1}{n}\le x\le 1 \end{cases}$$

Justify that $\lim\limits_{n\to\infty}\int_0^1 f_n \neq \int_0^1 \lim\limits_{n\to\infty} f_n$.

iii. Prove that the Riemann's Integral of a function if exists is uniquely determined.

iv. Discuss point-wise and uniform convergence of $f_n:(-\infty,+\infty)\to(-\infty,+\infty)$ defined as $f_n(x)=x^n$.

v. Prove that the sum of two Riemann integrable functions is Riemann integrable.

vi. Prove that every Riemann's Integrable function is bounded. Give an example to show that converse of this statement does not hold.

Section III (Essay Type)

Answer the following Questions 4x6 = 24

Q.no.3 Define Daurbox integral and prove that a function is Daurbox integrable if and only if it is Riemann integrable.

Q.no.4 Prove that every continuous function defined on closed interval is Riemann integrable. What about its converse.

Q.no.5 Let (f_n) be a sequence of bounded functions on $A\subseteq R$. Then prove that this sequence converges uniformly to a function f if and only if for each $\varepsilon>0$ there is a number $H(\varepsilon)$ in N such that for all $m,n\ge H(\varepsilon)$, $\|f_m-f_n\|<\varepsilon$.

Q.no.6 Let $g:[0,3]\to(-\infty,+\infty)$ be a function defined as

$$g(x)=\begin{cases} 3 & \text{for } 0\le x\le 1, \\ 4 & \text{for } 1<x\le 3. \end{cases}$$

Find its Riemann integral by using definition.